RAL-TR-96-028
hep-ph/9604413

GAUGE UNIFICATION AT THE STRING SCALE AND FERMION MASSES

B.C.Allanach[1], Rutherford Appleton Lab, Didcot, Oxon., OX11 OQX, UK.
S.F.King, Dept. of Physics, University of Southampton, Southampton, SO17 1BJ, UK.

In the context of the minimal supersymmetric standard model (MSSM), we discuss the introduction of exotic matter below the string scale M_X in order to achieve gauge unification at M_X (a constraint of a large class of string models). The possible types of exotic matter that can realise this are investigated and its effect on the top quark mass m_t is presented. The implementation of a theory of fermion masses which utilises the exotic matter is briefly discussed.

[1]Talk presented by B.C.Allanach at *XXXI Rencontres de Moriond.*

Gauge Unification

It has been known for a long time that a constraint of simple GUTs such as SU(5), SO(10) is that the gauge couplings should become equal when evolved up to a high energy scale. This scale is the energy at which the GUT breaks down into the Standard Model M_{GUT} and this "unification" means that

$$\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT}), \tag{1}$$

where $\alpha_{1,2,3}$ are the gauge couplings corresponding to the $\mathrm{U}(1)_Y$[2], $\mathrm{SU}(2)_L$ and SU(3) simple gauge groups in the Standard Model (SM) respectively. Several authors have shown that Eq. 1 is not satisfied when the effective theory at energy scales between M_{GUT} and M_Z is the SM. The gauge couplings evolve because the gauge boson propagator has a divergent contribution coming from loops of particles with a mass less than the relevant energy scale. This means that if the particle content of a theory is changed, so is the evolution of the gauge couplings. When the SM is supersymmetrised to become the MSSM, the extra particles ("superpartners" and Higgs) alter the evolution in such a way that the values of $\alpha_i(M_Z)$ that are extracted from experiment and evolved to M_{GUT} satisfy Eq. 1 to a good accuracy[1)]. This point is illustrated by Fig. 1 in which the solid lines show the gauge couplings meeting at $M_{GUT} \sim 10^{16}$ GeV. Simple GUTs only have one gauge coupling, which then evolves at scales $\mu > M_{GUT}$ as shown[3]. While GUTs provide a simple and elegant scheme for helping to explain the origin of the strong and electroweak forces, they do not include any quantum description of gravity. The only known consistent theories of quantum gravity to date are superstring theories, and we now turn to these to examine how the above apparent success of gauge coupling unification in GUTs translates into string models.

String Gauge Boundary Conditions

In string models, the constraints on the low energy[4] gauge couplings may

[2] α_1 in Eq. 1 assumes the GUT normalisation of the hypercharge $Y^{GUT} = \sqrt{3/5}Y^{SM}$.

[3] In the figure, the running is only shown from $\mu = M_{SUSY} \sim 1$ TeV but one should bear in mind that the input values at M_{SUSY} are the ones extracted from experiment and evolved to M_{SUSY} using the Standard Model renormalisation group.

[4] Low energy in this paper refers to anything less than the string scale M_X.

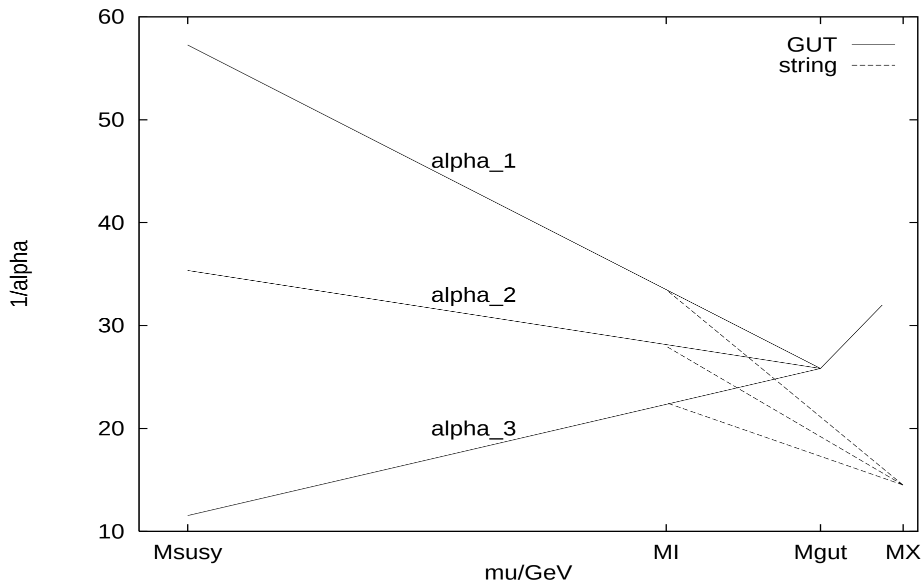

Figure 1: Evolution of the gauge couplings with energy scale μ in the context of a supersymmetric GUT model and a string model with $k_1 = 5/3$ and extra matter added with mass M_I. The extra matter in this case is $2 \times (3, 2)$ and $3 \times (3, 1)$ representations (and antiparticles).

be different to the ones in Eq. 1. In fact, the relevant relations are[2)]

$$\frac{3}{5}k_1\alpha_1(M_X) = k_2\alpha_2(M_X) = k_3\alpha_3(M_X) \equiv \alpha_{string} \tag{2}$$

where k_i are the Kac-Moody levels associated with the string model. For our analysis, the important fact about k_i is that they are numerical constants that are set, once the string model has been chosen. For non-abelian groups, the Kac-Moody levels must be natural numbers whereas they are rational for abelian groups[3)]. This means that $k_{2,3} \in \{1, 2, 3, \ldots\}$, $k_1 \in Z/Z$. The only semi-realistic models (i.e. with 3 families etc.) that have been constructed to date have $k_{2,3} = 1$ and so we concentrate on this case.

At first sight it appears that one constraint is lost compared to the GUT case because we have imposed no conditions upon k_1, the normalisation of Y. However, string models with $k_{2,3} = 1$ relate the scale of the breakdown of string theory to the gauge couplings through the relation

$$M_X = 5.3\sqrt{4\pi\alpha_{string}} \times 10^{17} \text{ GeV}. \tag{3}$$

This would mean that low energy phenomenology is incompatible with the prediction $\alpha_2(M_X) = \alpha_3(M_X)$, since if the gauge couplings were evolved to M_X in the MSSM, the solid lines in Fig. 1 would still cross at M_{GUT} and be different at M_X. There are, however several reasons[4)] why the measured gauge couplings might (wrongly) appear to be in conflict with Eq. 2 and therefore with the class of string models that we are advocating. Several of these possible reasons have been discussed by other authors[4)], but we turn to one particular possibility that unambiguously satisfies Eq. 1.

Intermediate Matter

One can imagine a theory in which some matter additional to the MSSM[3),4),5)] has a mass M_I where $M_Z < M_I < M_X$. Below M_I we have the MSSM, and above it the effect of the extra matter is felt upon the gauge couplings. In string models which break to the MSSM and have $k_2 = k_3 = 1$, the only possible matter fields present in the low energy theory are (3,1), (1,2) and (3,2) representations[5]. Leaving hypercharge assignments aside, these fields look like extra copies of right handed quarks q_R, left handed lepton doublets

[5] Written in (SU(3),SU(2)$_L$) space.

L_L and left handed quarks Q_L respectively[6]. We label the number of q_R, L_L and Q_L fields a, b, c respectively. One can solve $\alpha_2(M_X) = \alpha_3(M_X)$ and Eq. 3 to obtain $M_I, M_X, \alpha_{string}$ for each choice of possible intermediate matter. In Fig. 1, the dotted lines show how the intermediate matter can "re-focus" the gauge couplings to meet at the string scale M_X. The bound $M_I < M_X$ implies that

$$a > b + c \tag{4}$$

for the model to unify $\alpha_2(M_X) = \alpha_3(M_X)$.

Top Quark Mass

When one evolves the top quark Yukawa coupling h_t from high to low energy scales in the MSSM, one finds that the RGEs naturally "focus" the couplings into a narrow range at low energy centred around the infra red stable fixed point (IRSFP)[6)]. This IRSFP of h_t in the MSSM corresponds to $m_t/\sin\beta \approx 174 - 195$ GeV[7]. The "quasi fixed point" (QFP) is defined when h_t at some high energy scale (say M_X) is large, and in this case the low energy value of the top mass $m_t/\sin\beta \sim 210$ GeV is independent of what the actual value of $h_t(M_X)$ is. The IRSFP of the top quark Yukawa coupling is described analytically by

$$\left(\frac{h_t^2}{4\pi\alpha_3}\right)^* \sim (16/3 + b_3)/6, \tag{5}$$

where b_3 is the QCD beta function. In the region of energy scales between M_I and M_X, we have added coloured matter which changes b_3 and could possibly change the low energy prediction of the top quark mass. Overall, the effect of the intermediate matter is to drive h_t nearer to its QFP value that corresponds to $m_t/\sin\beta \approx 210$ GeV.

Lighter Fermion Masses

We now address the question of how the pattern and hierarchy of the fermion masses and mixing angles may arise. In our models, the only renor-

[6] It is to be understood that the supersymmetric partners and conjugate (antiparticle) fields are to be automatically included in the spectrum.

[7] $\tan\beta$ is the ratio of the two Higgs vacuum expectation values (VEVs) of the MSSM.

malisable fermion mass term is the one in the 33 element of the mass matrices, with all the others initially being zero. We now consider how the lighter fermions could acquire mass and mixing angles. One idea[7)] is to extend the gauge symmetry of the MSSM by a family dependent abelian $\mathrm{U}(1)_X$. The theory contains SM singlets $\theta^1, \bar{\theta}^{-1}$ which acquire VEVs, and these break the $\mathrm{U}(1)_X$ symmetry. We introduce some heavy Higgs doublets of mass M with different X charges and these help to generate non-renormalisable operators which play the role of mass and mixing terms. The other entries in the matrix appear once θ acquires a VEV $\langle\theta\rangle$. The nonrenormalisable operators produced in this way can generate a predictive and explanatory scheme of masses and mixings angles[7),8)].

However, we have already been using the candidate heavy Higgs to help unify α_i in string scenarios so we may be able to put these to work to give a predictive theory of fermion masses. In this framework, the $\mathrm{U}(1)_X$ has mixed anomalies with SU(3), $\mathrm{SU}(2)_L$, $\mathrm{U}(1)_Y$ and to cancel these using a (string-type) mechanism called GSW, the Kac-Moody levels must be in the ratio

$$k_1 : k_2 : k_3 = 5/3 : 1 : 1. \tag{6}$$

The GSW mechanism also requires $\langle\theta\rangle/M_X \sim O(1/40)$ but for a correct fermion mass hierarchy we require $\langle\theta\rangle/M_I \sim 0.2$. Thus there is an extra constraint on the models that

$$\frac{M_I}{M_X} = O(1/8). \tag{7}$$

We may now search through the models to see if any can make the gauge couplings unify at M_X *and* give an explanatory and predictive theory of fermion masses. This may be achieved by checking that Eq.s 7,2,6 are satisfied and that the extra matter is in a form to give the correct pattern of light fermion masses.

Summary

Intermediate matter is an effective way of obtaining gauge unification at the string scale. Its mass may come from hidden sector dynamics or non-renormalisable string-type operators[9)]. The intermediate matter can be combined with an abelian family dependent gauge symmetry to yield the

lighter fermion masses. It is now possible to construct[10] explicit models which incorporate both a predictive and explanatory theory of fermion masses and which unify the gauge couplings at the string scale. In this manner, we may develop a realistic theory coming from a superstring model.

References

[1] U.Amaldi et al., Phys. Rev. **D36** (1987) 1385.

[2] J.Ellis, S.Kelly and D.V.Nanopoulos, Phys. Lett. **B249** (1990) 241.

[3] S.Martin and P.Ramond, Phys. Rev. **D51** (1995) 6515.

[4] K.R.Dienes and A.E.Faraggi, Nucl. Phys. **B457** (1995) 409.

[5] B.C.Allanach and S.F.King, hep-ph/9601391.

[6] M.Lanzagorta and G.G.Ross, Phys. Lett. **B349** (1995) 319.

[7] L. Ibanez and G.Ross, Phys. Lett. **B349** (1994) 100.

[8] G.G.Ross, Phys. Lett. **B364** (1995) 216.

[9] G.Leontaris, Phys. Lett. **B281** (1992) 54.

[10] B.C.Allanach and S.F.King, work in progress.